Sharpness, Resolution and Interpolation

Introduction

There are a lot of misconceptions about resolution, camera pixel count, interpolation and their effect on astronomical images. Some of the confusion revolves around the difference between image sharpness and resolution. In this paper we will examine each and show that pixel count is not a guarantee of image sharpness or high resolution.

A Few Concepts

To understand all of these things we need to understand a few concepts. Cameras spatial sample an image and the same theories that are used to describe sampling an audio signal in time apply to sampled images. The sample rate for an audio signal is described in terms of samples per second while for images can be thought of as samples per inch or samples per image. Just like in audio sampling there is a limit to what frequencies can be sampled at a given sample rate. Here the frequencies are spatial frequencies and are a measure of how fast the value can change pixel to pixel. The highest frequency that can be represented in any sampled system is equal to one half the sample rate. For images this means that some image detail spans two pixels. For stars, one pixel is on a star and the very next pixel is on the background.

What is resolution and sharpness?

Many feel that resolution is what produces image sharpness, but that is only partially true. Image sharpness is really an indication of well balanced data in the spatial frequency domain no matter the size of the image. Resolution is a measure of how many pixels cover a given detail in an image. First let's take a look at resolution and sampling using my imaging system as an example. I use an eight inch SkyWatcher imaging Newtonian with a Paracorr coma corrector and collect the photons on a Canon 60Da.

When light passes through a circular lens and comes to a focus it produces an Airy disk. Many of you are familiar with the diffraction pattern as you see it each time you look at a magnified star in a high power eyepiece.



Figure 1 - Airy disk

Figure 1 shows the typical view of an Airy disk as seen in a telescope. Two stars are considered resolved when the maximum of the central spot of one star lies on top of the first minimum in the Airy disk of the other star as shown in the plot below.

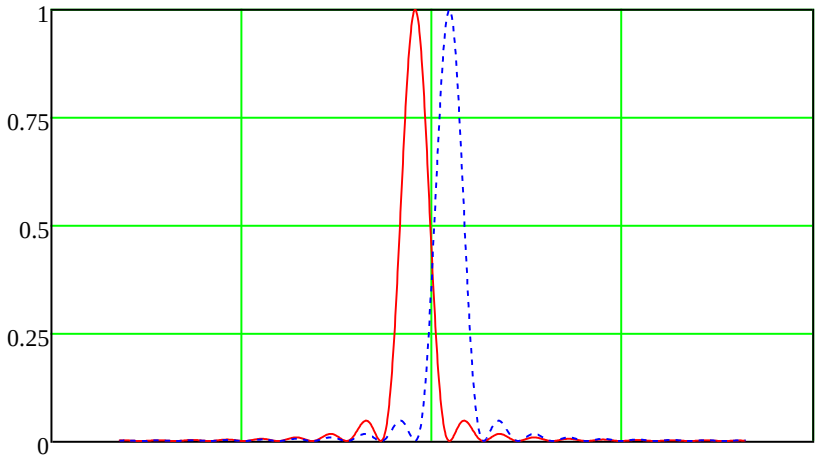

Figure 2 - Two stars just resolved

This is the absolute minimum separation that two stars can have and be detectable. Note that the two stars will look like an elongated star, not two separate stars.

The distance on the focal plane is $x = 1.22 * \frac{\lambda * f}{d}$ where λ is the wavelength, f is the focal length and d is the lens diameter. If we use 559 nm for the wavelength (the center of the visible light spectrum) and substitute the focal ratio (F) for f/d then we have, $x = 681.98\ nm * F$. For my scope (f/5.75) the resolution limit is 3.9 µm at the focal plane. Now with a focal length of 1150 mm my optics yield a theoretical resolution of 0.7 arc-seconds calculated from $\theta = 2 * \text{atan}\left(\frac{3.9\ um}{2 * f}\right)$, where f is the scope focal length. Keep in mind that this is the theoretical best possible resolution for my system assuming

perfect optics and observing in a vacuum. Seeing on average is between one and two arc-seconds, call it 1.5 arc-seconds which is about half the best I can expect with my optics and thus places the real limit on my optical system.

My camera employs a sensor with 4.3 µm pixels producing an image scale of 0.77 arc-seconds per pixel with my optics. But, since it is a DSLR with an anti-alias filter that slightly blurs the image (let's assume over two pixels) then the resolution is about 1.6 arc-seconds. This assumes that the camera completely compensates for the effect of the Bayer matrix, which of course it does not. With a resolution of around 1.6 arc-seconds when coupled with my optics, it is clear that my camera and seeing place the limit on the resolution of my imaging system. The above discussion does not include any effect from the Bayer matrix and assumes that the demosaicing required may cause colour bleed but does not affect resolution. While this is not strictly true, all image elements are sampled in at least one colour so demosaicing should be able to maintain a reasonable representation of the luminance of the image.

With a resolution of 1.4 arc-seconds and seeing of about 1.5 arc-seconds, I should be just able to resolve Epsilon Lyrae where the pairs are just over two arc-seconds apart. As you can see from the image in Figure 3, the resolution is just about exactly what is predicted from the math with the pairs just resolved.

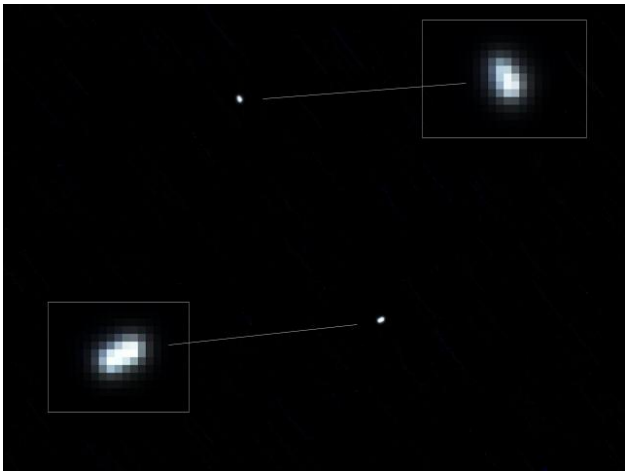

Figure 3 - Epsilon Lyrae imaged at prime focus with my imaging system

With a separation of 2.3 arc-seconds for the upper pair and 2.6 arc-seconds for the lower the stars are very close to the resolution limit of my system. The stars blend together and form an extended object at the image plane and are just resolved. Now from the image you can clearly see how the light from the stars is spread out over several pixels. You can also see that the change in brightness takes place over about four pixels from the stellar core to the background indicating that the highest spatial frequency present in the image is at least half the Nyquist frequency.

The above discussion shows that resolution is not just a function of the number of pixels in the imaging system. Everything in the optical chain, including the atmosphere plays a part in determining the overall system resolution. After all, we put observatories on mountain tops for a reason.

Now that we have a working definition of resolution, *the smallest separation between two picture elements that can be discerned*, we can take a look at the much more subjective concept of image sharpness. To borrow a phrase – sharpness is one of those things you'll know when you see it. Sharpness and resolution may be linked, but sharpness and pixel count are not. It is entirely possible to have a high pixel count yet blurry image. What we need is some way to empirically measure image sharpness. Let's take a look at a few images, the first is a small image produced mathematically and contains vertical stripes. The other images are interpolated from this image to increase the pixel count then cropped so they can be displayed at 100 percent.

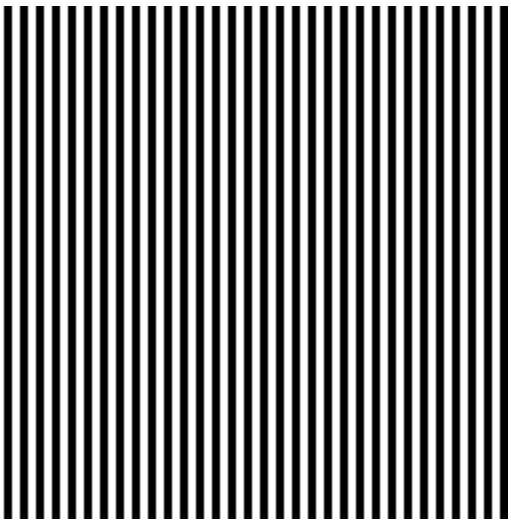

Figure 4 - Small striped image

The original image in Figure 4 is sharp with well defined edges in the transition from the black to the white stripes. The next image is produced by using interpolation to increase the image size by a factor of two; a bi-cubic interpolation filter was used. Below is a 100 % crop of a section of the interpolated data.

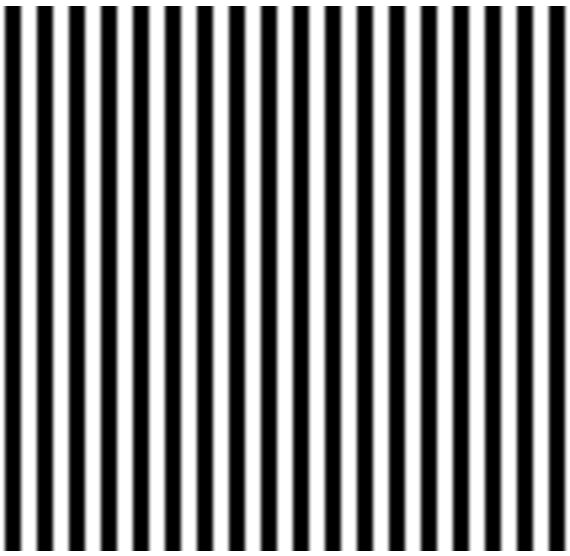

Figure 5 - Image interpolated by two

Note how the transition from black to white is not quite as sharp as the original. Finally the last image is a 100 % crop of the initial image interpolated by four.

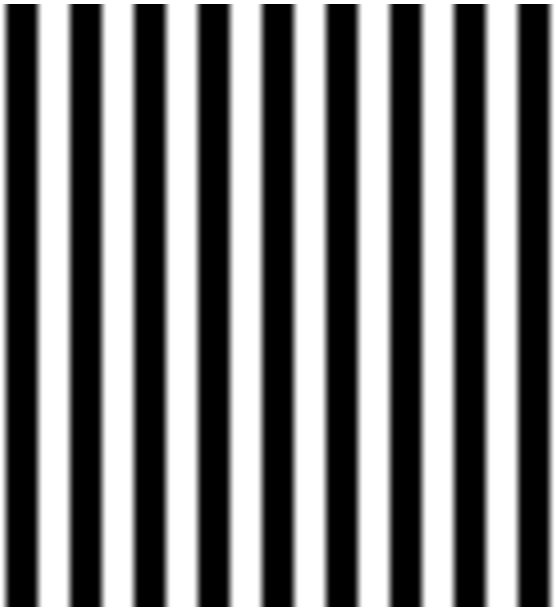

Figure 6 - Image interpolated by four

If you closely examine each striped image you will notice that as the image size grows, the lines become less sharp. Now the question becomes, is there some measurement that we can use to judge image sharpness? The answer to this lies in the spatial frequency spectrum of each of the images.

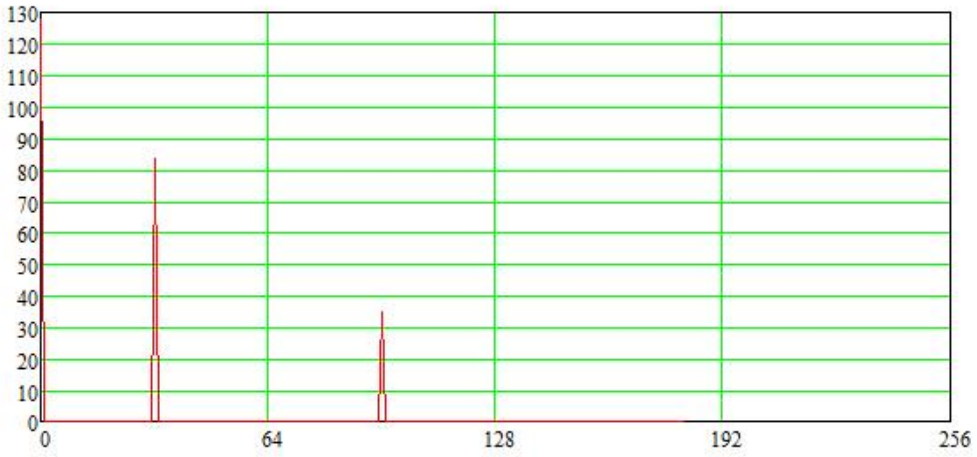

Figure 7 – Spectrum of the small striped image

Examining the spectrum of the small image, Figure 7, we see that the Nyquist frequency is 128 and that the highest frequency contained in the image is close to Nyquist at 96. Dividing the Nyquist frequency by the highest frequency of significant level gives us 128/96 or a ratio of 1.3. The frequency scale here is

somewhat arbitrary and is simply the number of pixels from the center of the 2D spectrum. Now let's look at the spectrum of the slightly fuzzier image that has been interpolated by a factor of two.

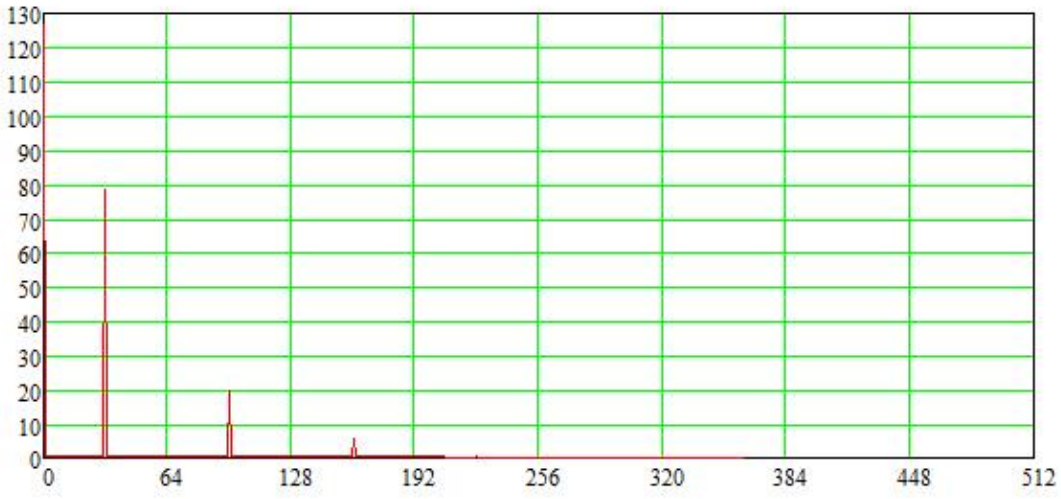

Figure 8 - Spectrum of image interpolated by two

Here the Nyquist frequency is higher at 256, the ratio is 1.6 and the actual image is slightly blurrier than the original. Finally, examine the spectrum of the image that has been interpolated by four.

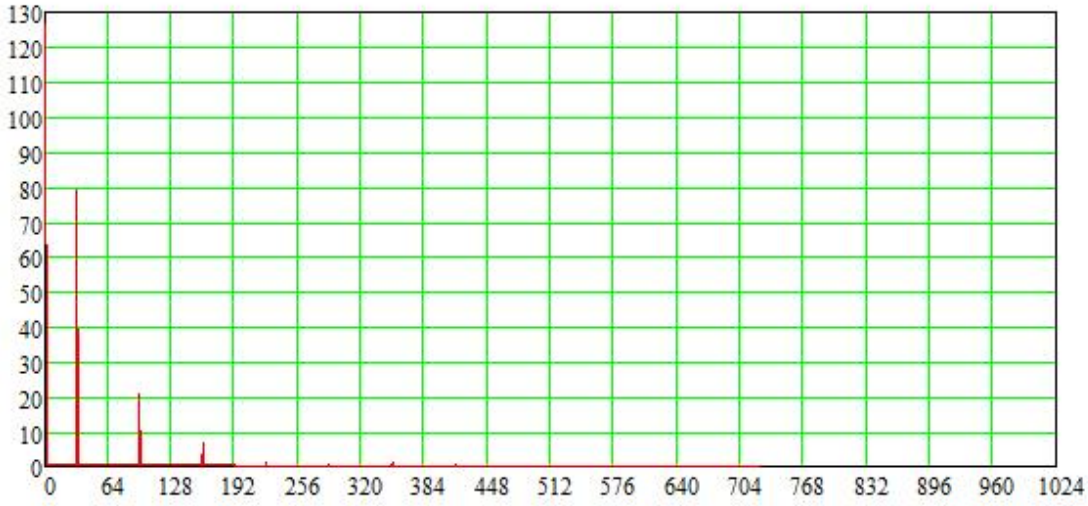

Figure 9 - Spectrum of image interpolated by four

Here the ratio is 512/160 or 3.2. There are spectral components above 160, but they are only about one percent of the main peak and of little impact on the image. Comparing the interpolated image with one that was drawn at the same scale shows the interpolated image is somewhat blurrier than the one drawn at full pixel count.

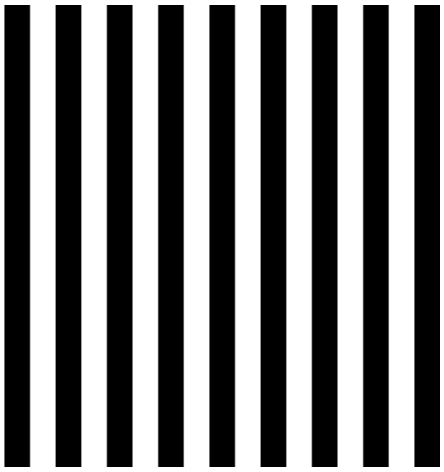

Figure 10 - Non-interpolated image

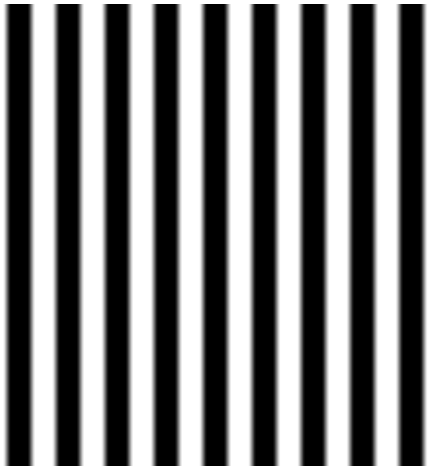

Figure 11 - Interpolated image

The image in Figure 10 shows better edges and is a generally sharper version than the one shown in Figure 11. Examining the spectra of both images, Figure 9 and Figure 12, shows that the ratio of the highest frequencies to the Nyquist frequency is very different for both images. While the ratio for the image in Figure 9 is 3.2, the ratio for the image in Figure 12 is 1.06 with significant frequency content out to 480 as shown below.

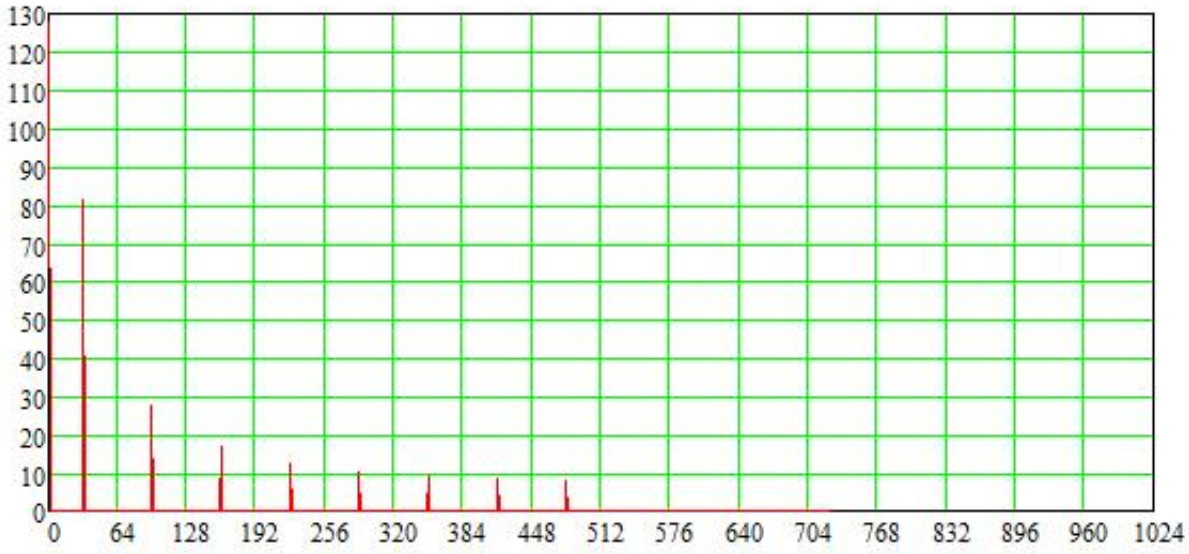

Figure 12 - Non-interpolated image spectrum.

This points to a simple rule for judging image sharpness, the closer the highest frequency data, excluding noise, is to the Nyquist frequency, the sharper the image. The simple rule for evaluating image sharpness *is the higher the ratio, the fuzzier the image*.

Here are two versions of a M20 image, the one on the right has been sharpened using a high pass filter.

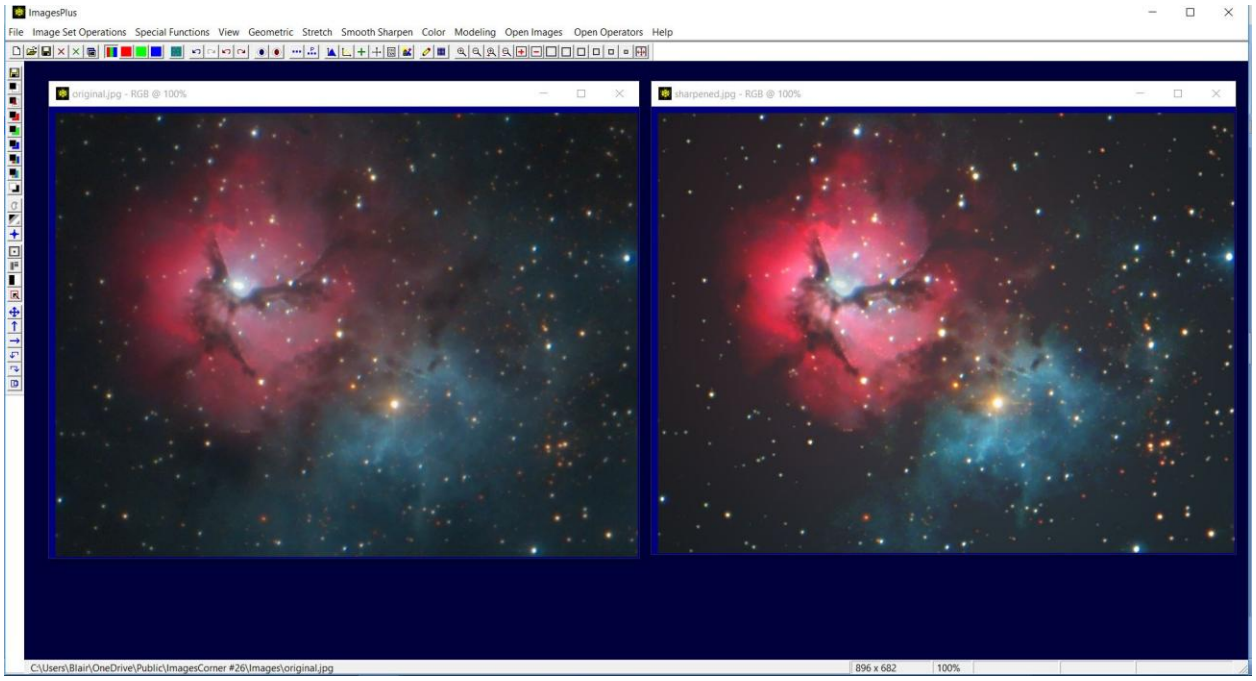

Figure 13 - Original image on left, sharpened version on the right

Now let's examine the spectra of the images to see what differences we see in their frequency content.

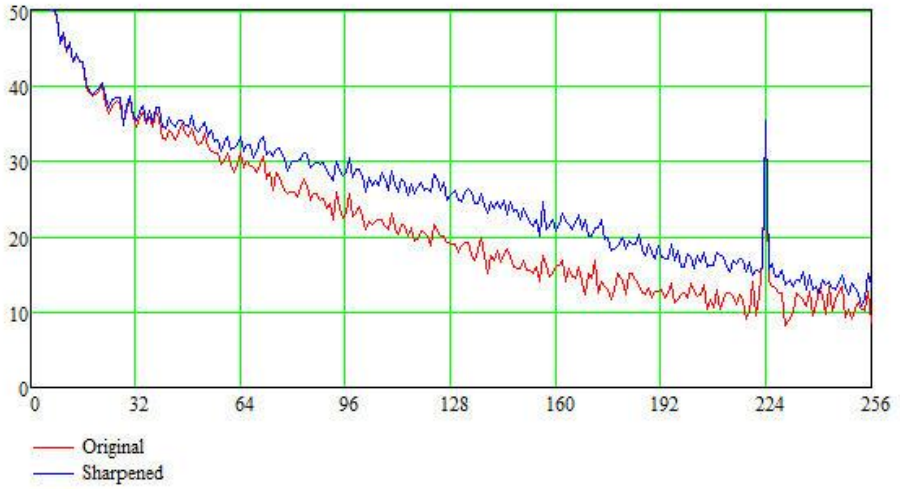

Figure 14 - Sharpened versus original M20 spectra. The data has been converted to dB (20*log(data)) to make the low level data more obvious.

The spectral data in Figure 14 clearly shows that the ratio rule developed using simple striped images holds for real images as well. The sharpened image has more high frequency content as the plot approaches Nyquist (256 for these images).

The reason that interpolated images look blurrier than the original is explained by the image spectrum as well. Although interpolation increases the pixel count it cannot make spatial frequency components that were not in the original data. As we add more pixels to an image the Nyquist frequency climbs, but the highest significant frequency doesn't change so the ratio rises resulting in a blurrier image. Much of the missing sharpness in interpolated images can be restored by simple sharpening using deconvolution. This changes the relative balance between the low frequency components and high frequency edges making for a clearer image. The two M101 images below show what good interpolation can do when you keep the spectrum in mind as you process.



Figure 15 – 100 percent crop of a full size M101 image



Figure 16 - Image made from a binned version of the original then interpolated and sharpened

Figure 16 was first binned by two then interpolated and sharpened to produce an image the same size as the original. Since the data was collected with my imaging setup, binning by two does not remove any data as the spectral components near Nyquist simply are not there to begin with. This is because the resolution of my system is about half what is required to produce spectral components at Nyquist. Since no image data is lost in the binning due to the limited resolution of my system, interpolation is able to produce an image very close to the original after just a little sharpening. Binning the image reduces the Nyquist frequency of the image by a factor of two so the highest frequency data from my system is now near Nyquist for the binned image and interpolation will faithfully reproduce the original image. These kinds of results are only available by knowing the true resolution of your imaging system and selecting a binning size that respects the spectral content of the original image.

A lot of DSLR's are limited in resolution to about half the Nyquist spatial frequency due to the blurring effects of their anti-alias filter. Depending on your equipment and seeing you may have a similar limit set by the atmosphere and your optics. If this is the case for your imaging system, then feel free to bin the image by two knowing that you will not lose any real data and that you can restore the original image with very little error using simple interpolation and a little sharpening.